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Number density weighted expression for thermal conductivity for ionospheric neutral particles

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Abstract : A number density weighted expression for thermal conductivity (K_n) for neutral particles in the ionosphere has been deduced using data from the models MSIS-83 and USSA-76 and the values of K_n for this have been compared with those from other relations

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The heating of the ionospheric ion gas by electron-ion elastic collisions and the loss of ion thermal energy in collision with neutral particles are the factors responsible for the distortion of the ion velocity distribution function from being Maxwellian. This distortion leads to cooling of the medium. The effectiveness of the neutral atmosphere acting as an ion heat sink depends upon the extent to which the neutral gas temperature is determined by ion-neutral energy transfer. As mentioned by Banks [1], since $\frac{3}{2}\, n(n)kT_n/L_t \gg 1$, the effective time for ions to heat the neutral atmosphere is large compared to the typical time for conduction cooling of the neutral atmosphere as pointed out by Nicolet [2]. Studies on thermal conductivity was made by several workers. Misra and Das [3] have also studied the same for ions, electrons and neutrals.

It is worth mentioning that an expression for thermal conductivity of the particles existing in a complicated thermal system, is difficult to obtain. Also, both the ions and neutrals of the ionosphere have effectively different thermal conductivities for each species. Hence, an effort has been made to make numerical analysis and to present an expression for appropriate number density weighted conductivity in the same way as Banks [1] did for ions. Of course, this relation may lead to errors of a few percent which is negligible for mixture of several species. The effects of ion-neutral collisions that tend to reduce the conductivity can be neglected for the present work. So far as neutral particles are concerned, the contributions of N_2, O_2, O, He and Ar are considerable, as pointed out by Hedin [4].

Straus *et al* [5] have given a relation for thermal conductivity of neutral particles (K_n) as

$$K_n = \sum_j \beta_j \frac{n_j}{n} T^{\alpha_j}, \tag{1}$$

where n_j is the neutral particle density of the j-th species in an ionospheric region having total neutral density n and temperature T. The coefficients α_j and β_j have been given according to Dalgarno and Smith [6] and Chapman and Cowling [7].

Again, Schunk [8] has presented an expression for K_n which is

$$K_n = \frac{75\sqrt{\pi}k(mkT)^{0.5}}{64m\pi\sigma^2}, \tag{2}$$

where k = Boltzman constant, m = mass of neutral particles, σ = sum of radii of colliding particles.

Schunk pointed out that by allowing for the appropriate collision processes (elastic collisions), eq. (2) could be applied to multicomponent plasma of arbitrary degree of ionisation as well as to a mixture of neutral gases. Moreover, processes like photo-ionisation, recombination and dissociation could also be considered. It has been assumed that collisions can occur between same species as well as with other particles. Considering hard sphere interactions, K_n becomes proportional to $T^{0.5}$ and inversely proportional to collision cross section [9]. The last one indicates that thermal conductivity is more important for atomic species like H, He and O than for molecular species like N_2, O_2 and NO *etc*.

Banks [1] has given a number density weighted expression for thermal conductivity of ions (K_i) as

$$K_i = \frac{1.2\times10^4}{n^+}\left[n^+(\mathrm{O}) + 2n^+(\mathrm{He}) + 4n^+(\mathrm{H})\right]T^{2.5}, \tag{3}$$

where $n^+(x)$ and n^+ are respectively the concentration of x ions and the total ions having temperature T. It is seen from eq. (3) that the conductivity K_i depends mainly on the ion densities and the temperature of the region concerned.

In this work, attempts have been made to provide an expression for thermal conductivity of neutrals in the altitude range of 90–220 km, as data in this region are available. Let us assume that the properties of a region remain constant over small horizontal slabs in which a temperature gradient exists in the vertical direction along which the variation of thermal conductivity has been considered. It is also assumed, for simplicity, that the effects of external fields like geomagnetic field and any type of electric field are absent.

Values of K_n have been computed from eq. (1) using data from the models MSIS-83 and USSA-76 and collision cross sections from Mott and Massey [9]. Considering these values, an expression for K_n, depending mainly upon particle densities and similar in form to eq. (3) for ions, has been framed. Of course, the parameters which affects K_n is a bit different from those considered in eq. (3). The expression could be written in the form

$$K_n = \left[a_1 n(\mathrm{N_2}) + a_2 n(\mathrm{O_2}) + a_3 n(\mathrm{O}) + a_4 n(\mathrm{He}) + a_5 n(\mathrm{Ar.})\right] \frac{T^{a_6}}{n}, \quad (4)$$

where $a_1, a_2, a_3, a_4, a_5, a_6$ are constants and $n(x)$ is the neutral particle density for x-th constituent, while n is the total number density of neutrals. The coefficients of the particle density as well as the power of T have been obtained by solving eq. (4) with calculated values of K_n from (1) and taking $n(x)$ and T from each of the models. To attain the goal K_n have been computed at an interval of 10 km in the altitude range from 90 to 220 km, both inclusive. From the relations for different altitudes (as mentioned above) obtained from (4) for each model and also for each of the eq. (1) for different altitudes, the constants a's have been found out as follows.

Let us put

$$Z = \frac{a_1 n(\mathrm{N_2}) + a_2 n(\mathrm{O_2}) + a_3 n(\mathrm{O}) + a_4 n(\mathrm{He}) + a_5 n(\mathrm{Ar})}{n}, \quad (5)$$

$m = a_6$ and $K_n = y$ in eq. (4). Then eq. (4) becomes $y = ZT^m$ or $y' = mx' + z'$, (6)

where $y' = \log y$; $z' = \log z$ and $x' = \log T$. Parameters like x', y' and z' in eq. (6) may change with altitude. It may be mentioned that altitude variation of z' is negligibly small. The main change occurs in the value of x', for that in y'. z' may be normalized to fit the data and a mean value may be taken. Taking T (*i.e.* x') from the models and computing K_n (*i.e.* z') from eq. (1) the value of z has been found to be 0.241. With this z, the value of m is found to be 0.5 for best fit. This agrees with the power of T in eq. (2). With $m = 0.5$, values of z have been computed from (1) which shows a slight variation with altitude as mentioned earlier. The variations of z' and z are shown in Figures 1(a) and 1(b). This means that the number density variation of neutral particles (however small it may be) has some

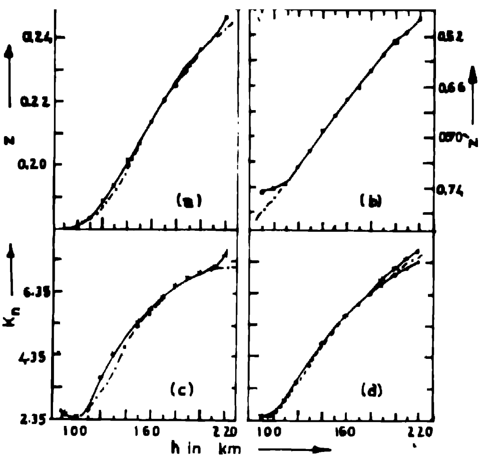

Figure 1. Altitude variation of z, z' and K_n (a) for z and (b) for z' Solid curves for MSIS-83 and dotted curve for USSA-76 models. (c) for K_n with data from MSIS-83 and (d) for K_n with data from USSA-76 models : (●) from eq. (1), (×) from eq. (8); dotted curve from eq. (2).

contribution towards the variation of K_n. It is well known that the particle density of a region depends mainly on its temperature. Hence, the variation of the later leads to the variation of the particle densities, collision mechanism *etc.*, thereby leading to variation of z.

Considering altitude variation of z due to variations in $n(x)$ and n, one may write

$$\begin{pmatrix} Z_1 \\ Z_2 \\ - \\ - \\ - \\ - \\ Z_{14} \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} & n_{13} & n_{14} & n_{15} \\ n_{21} & n_{22} & n_{23} & n_{24} & n_{25} \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ n_{14,1} & n_{14,2} & n_{14,3} & n_{14,4} & n_{14,5} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ - \\ - \\ - \\ - \\ a_5 \end{pmatrix}$$

i.e. $$(Z_i) = (n_{ij})(a_k)$$

or $$(a_k) = (n_{ij})^{-1}(Z_i), \tag{7}$$

where the suffix i and k denote the row number and j the column number of n_{ij}, i = 1, 2, 3, ..., 14 for altitudes 90, 100, 110, ..., 220 km while j and k = 1, 2, 3, 4, 5 for the relative abundance $(n(x)/n)$ of the species N_2, O_2, O, He and Ar respectively. Solving eq. (7), a's are found to be $a_1 = 1.52 \times 10^{-1} = a_2$, $a_3 = 3.04 \times 10^{-1}$, $a_4 = 3.344 = a_5$. These values of the coefficients are finally chosen by the methods of trial and error as well as rejection and acceptance to fit the computed values of K_n from eq. (1). This leads one to obtain the final form of the expression for K_n as

$$K_n = 1.52 \times 10^{-1}\left[n(\mathrm{N}_2) + n(\mathrm{O}_2) + 2n(\mathrm{O}) + 22n(\mathrm{He}) + 22n(\mathrm{Ar})\right] \frac{T^{0.5}}{n} \tag{8}$$

Values of K_n computed from eq. (8) have been compared to those from eqs. (1) and (2) for the models MSIS-83 and USSA-76 as shown in Figures 1(c) and 1(d) respectively. It is explicitly clear that these values agree well within small percentage of error. Hence, relation (8) may be taken as an expression for thermal conductivity for neutral particles considering the constituents N_2, O_2, O, He and Ar to be present in an ionospheric region.

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